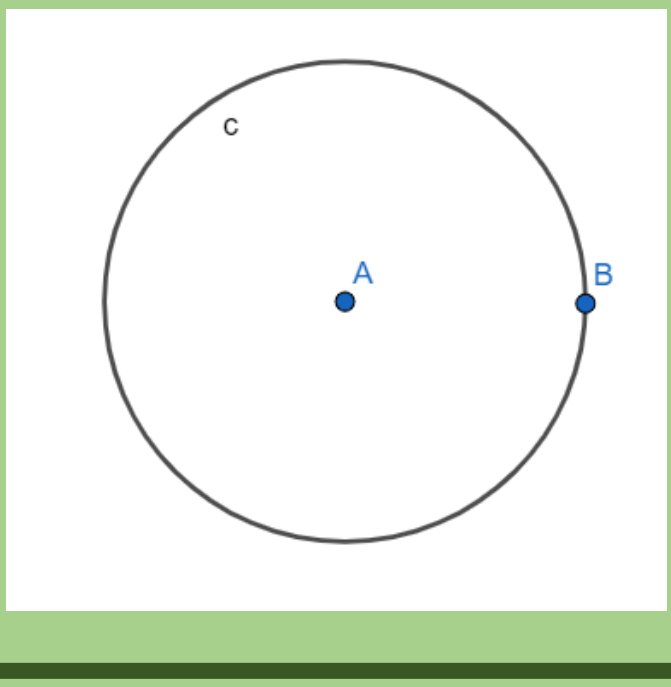
Fractals

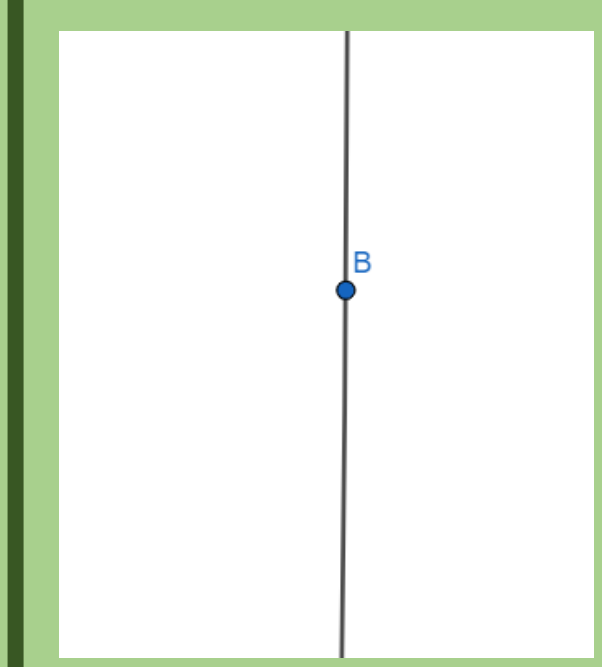
Introduction:

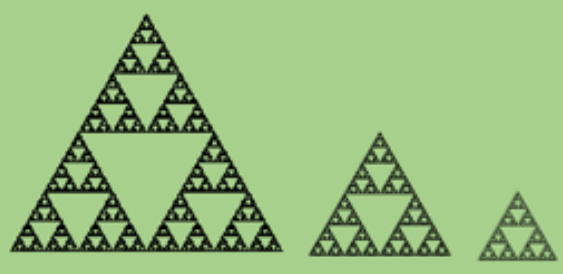
Fractals are a rather recent addition to the history of mathematics. The first ones were discovered towards the end of the 19th century and they brought a whole new dimension to geometry as we know it today. At the time of their discovery, they were labelled as pathologic objects. But far from being sickly, these objects are complex, beautiful and hypnotic.

What are fractals ?

Let us start by defining what they are not. If you consider a straight line, a curve, a broken line, a circle, or any other geometrical object that you might know, and zoom in multiple times on this object, after a few iterations, it will become flat and smooth. Let's observe this using GeoGebra.







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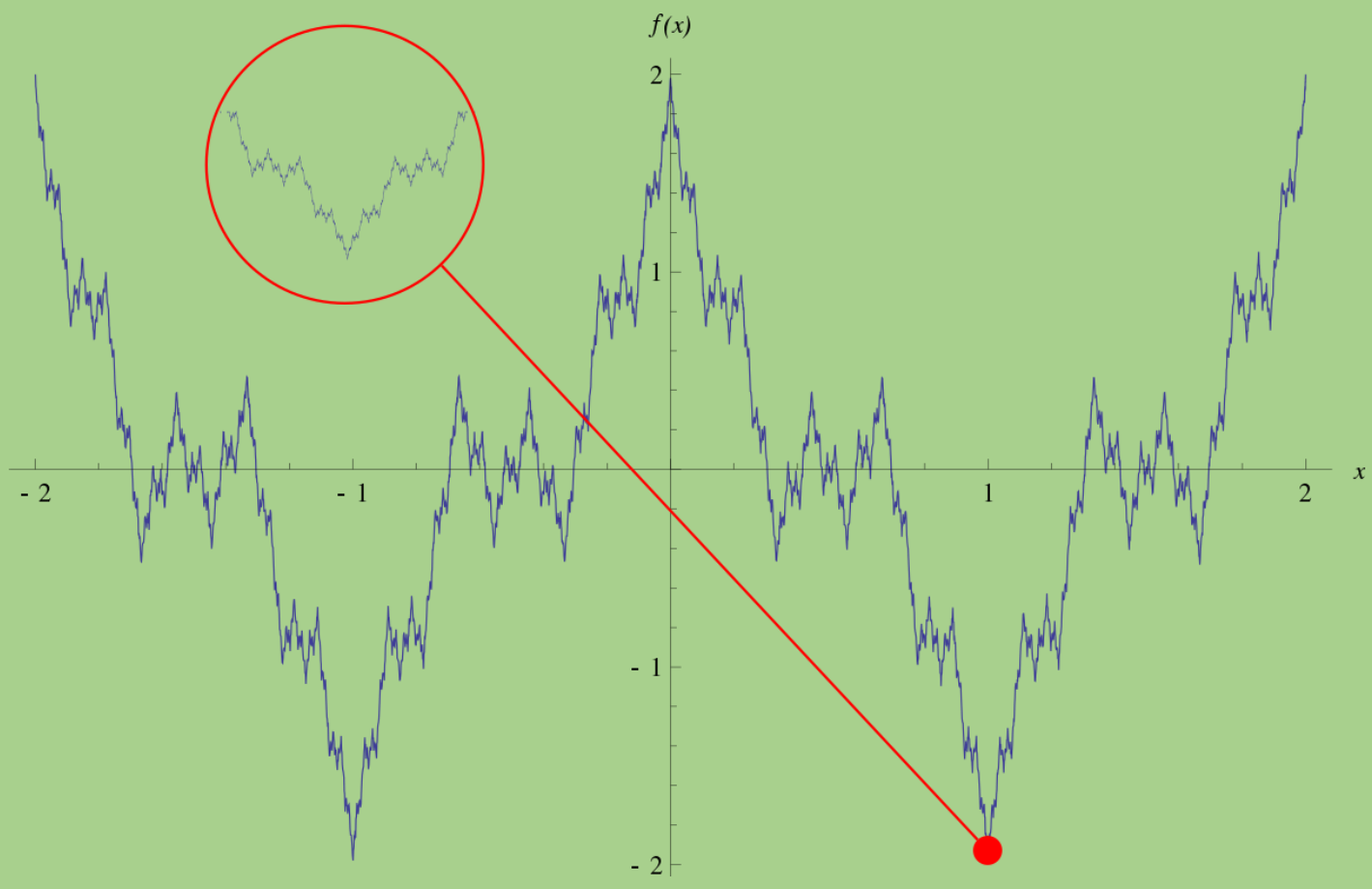
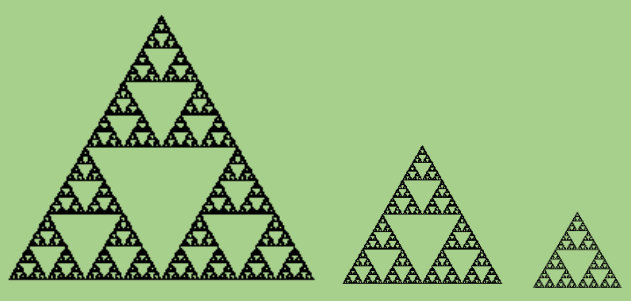
Interesting isn't it? Fractals, on the other hand, behave differently. Whatever the scale we choose, details in a fractal never stop to appear. To put it another way, these objects are « infinitely fragmented ». We can then understand the etymology of the word fractal which means « broken », « fragmented ». Look at this first fractal. It is called Weierstrass function and was discovered in 1861 by the mathematician bearing the same name.



Truth be told, this is not hugely aesthetic yet but one we can already see one of the fundamental concept present in fractals: auto-similarity. You might already understand this word intuitively. It means that we can find the global figure when looking at a smaller scale. The figure is a copy itself and this, infinitely. If you want to dive literally into the fractal universe, scan the QR codes below.



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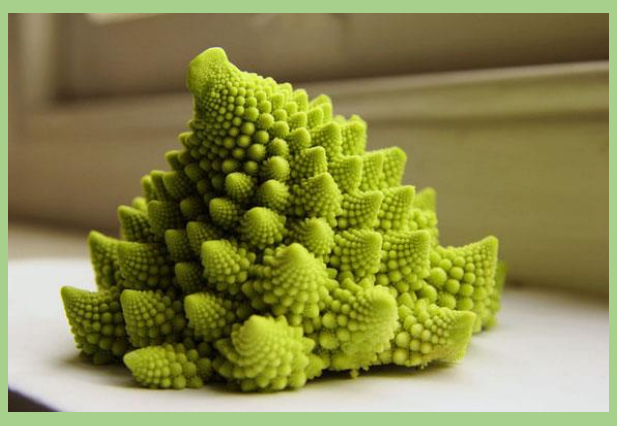
Fractals in Nature

What's even more fascinating with fractals is that they are not restricted to mathematics. We can also find them in nature.

For example, we can find fractals in a number of plants and animals :

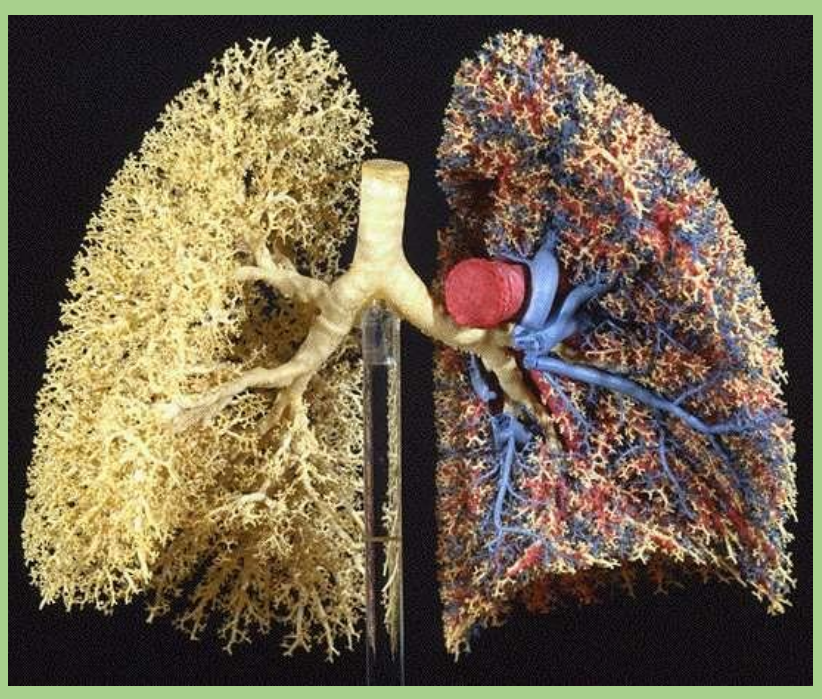


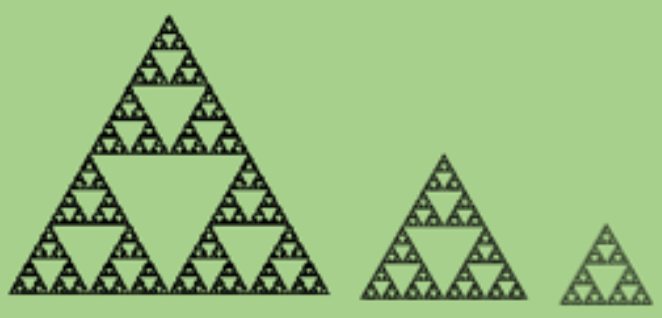
Here is The Romanesco cabbage

The Barnsley fern

Here is the tritoniopsis elegans

Or even nested in our chest cavity. As a matter of fact, our lungs apply the same principle as fractals by using numerous iterations of branches. This appears logical as our body is trying to maximise the surface in a fixed volume to increase the exchange of gas.





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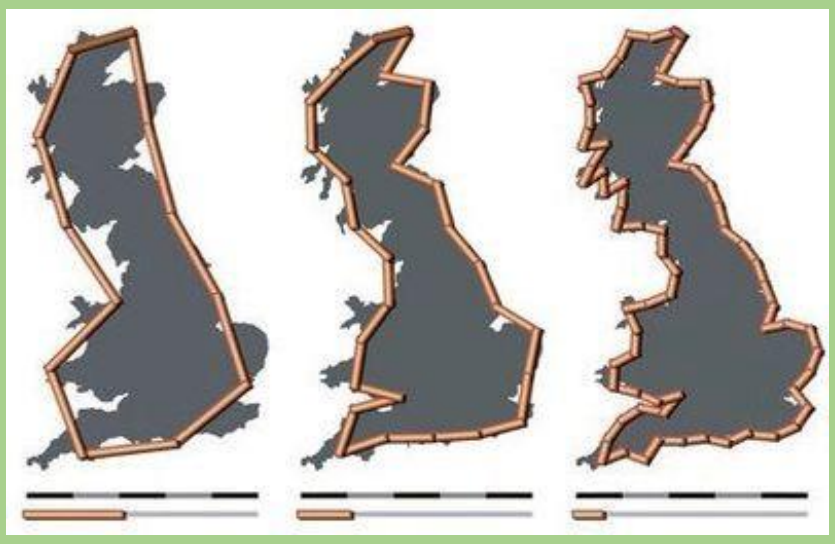
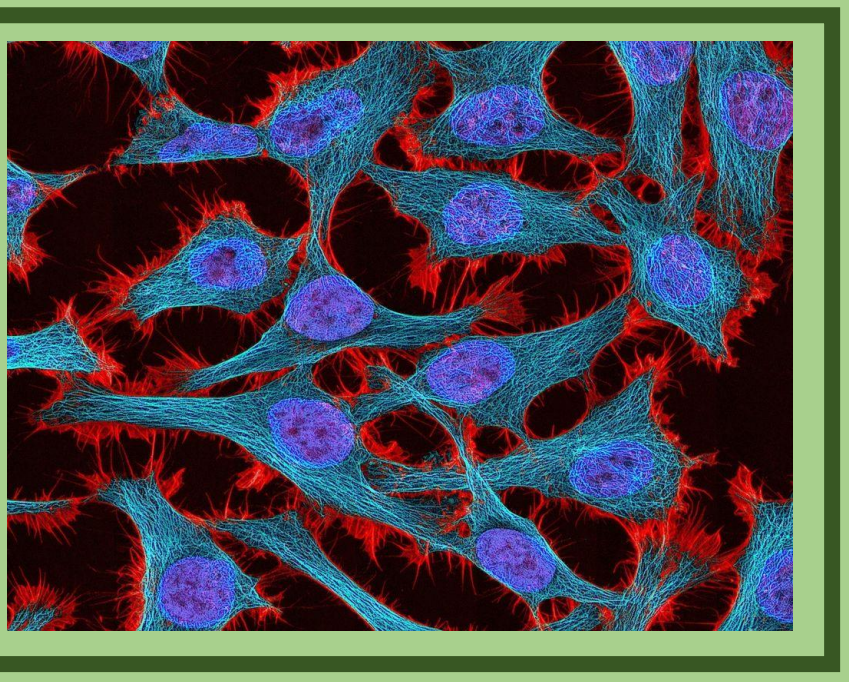
Fractals can also help calculate the length of a coastline. For example, I challenge you to measure Great Britain's coastline. Are you able to find it ? The task will quickly become difficult if you try to measure every rock, every grain of sand, or even every atom. But some mathematicians addressed this issue by applying fractal geometry and they calculated that its dimension was approximately 1.25.



There is a multitude of other applications of fractal geometry in nature, but let me show you one last one. Cancer cells are very rough and have a fractal dimension. By studying the « roughness » of cells using computers, it is possible to improve the detection rate of potential cancer cells. Those who called fractals « pathological » might have been right after all.



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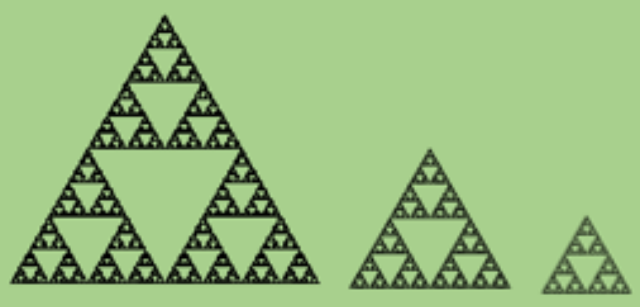


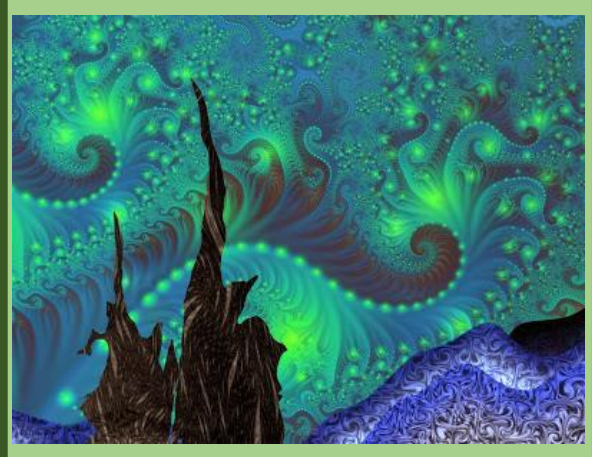
Fractales in Art

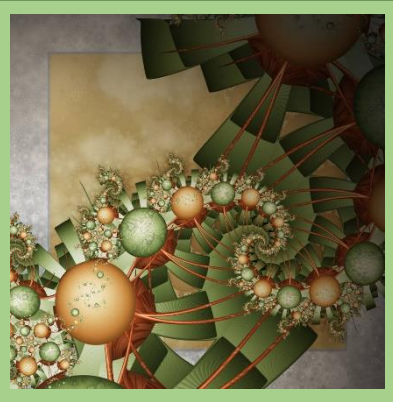
Do you think that art and mathematics are incompatible? Mandelbrot and his fractals will surely disagree with you. Art can even preceed mathematics sometimes. We can for example find fractal patterns in greek, roman, egyptian, celtic or islamic art...

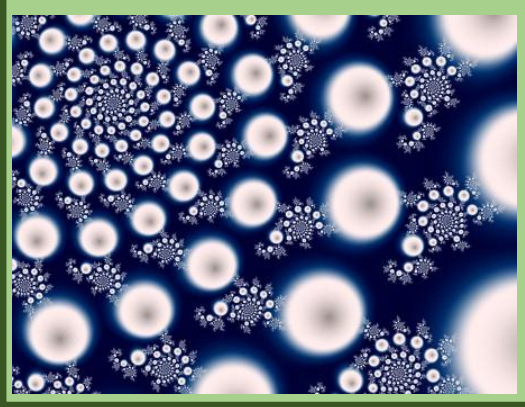
Desborough's mirror (1st century B.C.)

Roof of Selimiye's mosque (16th century)

But the progress made mainly by Mandelbrot in the study of fractals sparked a new interest for them. It even created a new type of art in the 1980s called « Fractal art ». It relies heavily on computers to generate drawings each one more breath-taking than the last.







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