Why study mathematics?

A talk delivered at French International School

Frederick K.S. Leung
Kintoy Professor in Mathematics Education
The University of Hong Kong
15 May 2017

Outline

1. Are Hong Kong students strong or weak in mathematics?
2. Why do so many students feel bored about mathematics?
3. What is the essence of mathematics? Is mathematics useful?
4. How can I support my child in learning mathematics?

Are Hong Kong students strong or weak in mathematics?

- What does research tell us?
- TIMSS: *Trends in International Mathematics and Science Study*
- Under the auspices of IEA (International Association for the Evaluation of Educational Achievement)
- Started in 1995 and repeated every four years: 1999, 2003, 2007, 2011, 2015, etc.
- Goal: "to provide comparative information about educational achievement across countries to improve teaching and learning in mathematics and science"

Countries participated in TIMSS 2015

Armenia
Australia
Bahrain
Belgium (Flemish)
Botswana
Bulgaria
Canada
Chile
Chinese Taipei
Croatia
Cyprus
Czech Republic
Denmark
Egypt
England
Finland
France
Georgia
Germany
Hong Kong SAR
Hungary
Indonesia
Iran, Islamic Rep. of
Ireland
Israel
Italy
Japan
Jordan
Kazakhstan
Korea, Rep. of
Kuwait
Lebanon
Lithuania
Malaysia
Malta
Morocco
Netherlands
New Zealand
Northern Ireland
Norway
Oman
Poland
Portugal
Qatar
Russian Federation
Saudi Arabia
Serbia
Singapore
Slovak Republic
Slovenia
South Africa
Spain
Sweden
Thailand
Turkey
United Arab Emirates
United States

Benchmarking Participants
Buenos Aires, Argentina
Ontario, Canada
Quebec, Canada
Abu Dhabi, UAE
Dubai, UAE
Florida, US

Results

Primary 4

Mathematics

Country	Average Scale Score
² Singapore	618 (3.8)
† Hong Kong SAR	615 (2.9)
Korea, Rep. of	608 (2.2)
Chinese Taipei	597 (1.9)
Japan	593 (2.0)
‡ Northern Ireland	570 (2.9)
Russian Federation	564 (3.4)
Norway (5)	549 (2.5)
Ireland	547 (2.1)
England	546 (2.8)
† Belgium (Flemish)	546 (2.1)
Kazakhstan	544 (4.5)
² Portugal	541 (2.2)
² † United States	539 (2.3)
² † Denmark	539 (2.7)
² Lithuania	535 (2.5)
Finland	535 (2.0)
Poland	535 (2.1)
† Netherlands	530 (1.7)
Hungary	529 (3.2)
Czech Republic	528 (2.2)
Bulgaria	524 (5.3)
Cyprus	523 (2.7)
Germany	522 (2.0)
Slovenia	520 (1.9)
² Sweden	519 (2.8)
³ Serbia	518 (3.5)
Australia	517 (3.1)
1 2 † Canada	511 (2.3)
² Italy	507 (2.6)
² Spain	505 (2.5)
Croatia	502 (1.8)
TIMSS Scale Centerpoint	500
Slovak Republic	498 (2.5)
New Zealand	491 (2.3)
France	488 (2.9)
Turkey	483 (3.1)
¹ Georgia	463 (3.6)
Chile	459 (2.4)
United Arab Emirates	452 (2.4)
² Bahrain	451 (1.6)
Qatar	439 (3.4)
Iran, Islamic Rep. of	431 (3.2)
Oman	425 (2.5)
Indonesia	397 (3.7)
Jordan	388 (3.1)
ψ Saudi Arabia	383 (4.1)
Morocco	377 (3.4)
South Africa (5)	376 (3.5)
ψ Kuwait	353 (4.6)

Mathematics Achievement Distribution

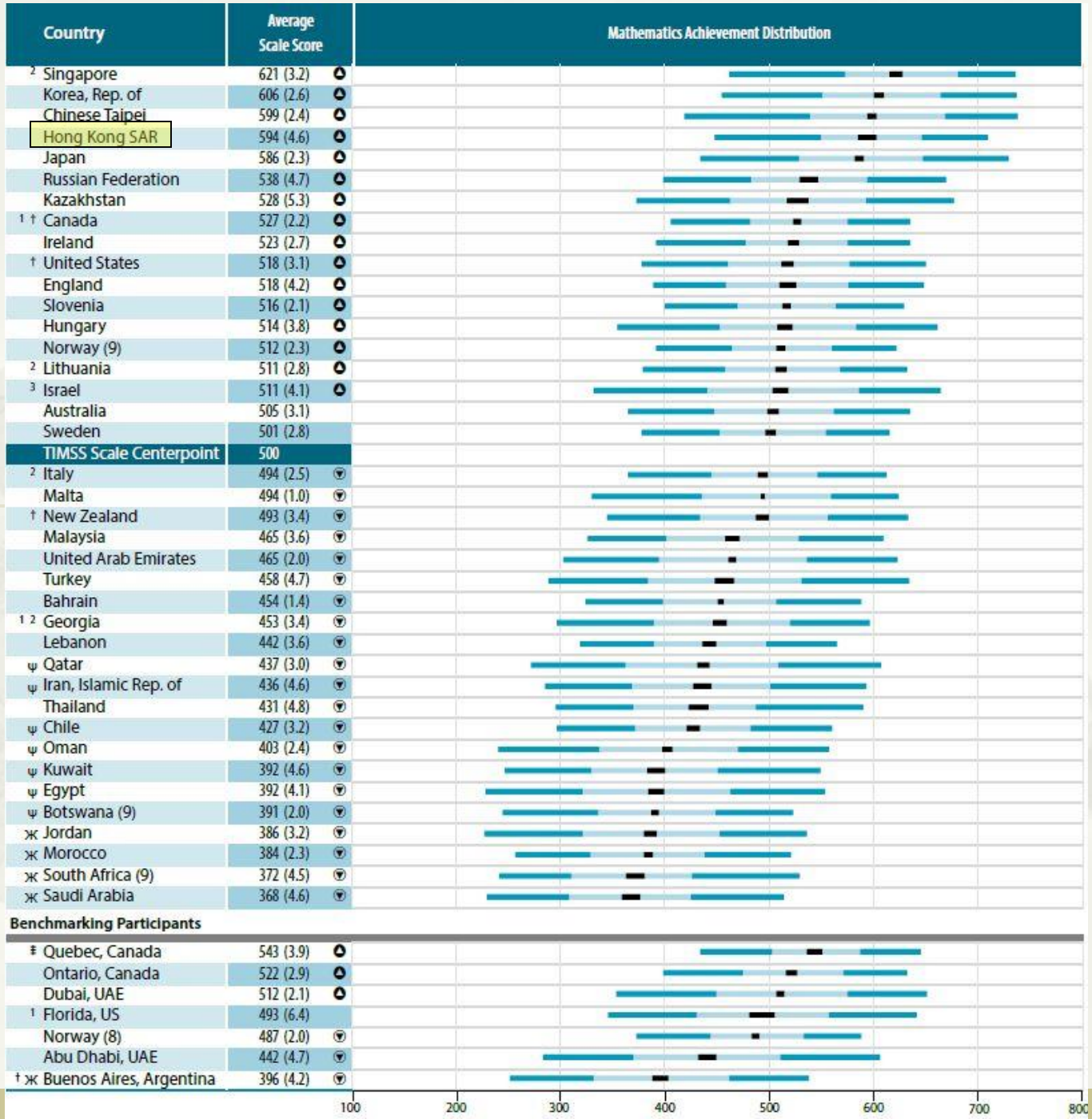
100 200 300 400 500 600 700 800

Primary 4 Mathematics (first 15 countries)

Country	Average Scale Score	Mathematics Achievement Distribution
2 Singapore	618 (3.8)	
† Hong Kong SAR	615 (2.9)	
Korea, Rep. of	608 (2.2)	
Chinese Taipei	597 (1.9)	
Japan	593 (2.0)	
‡ Northern Ireland	570 (2.9)	
Russian Federation	564 (3.4)	
Norway (5)	549 (2.5)	
Ireland	547 (2.1)	
England	546 (2.8)	
† Belgium (Flemish)	546 (2.1)	
Kazakhstan	544 (4.5)	
2 Portugal	541 (2.2)	
2 † United States	539 (2.3)	
2 † Denmark	539 (2.7)	

Secondary 2

Mathematics



Country	Average Scale Score
2 Singapore	621 (3.2) ▲
Korea, Rep. of	606 (2.6) ▲
Chinese Taipei	599 (2.4) ▲
Hong Kong SAR	594 (4.6) ▲
Japan	586 (2.3) ▲
Russian Federation	538 (4.7) ▲
Kazakhstan	528 (5.3) ▲
1 † Canada	527 (2.2) ▲
Ireland	523 (2.7) ▲
† United States	518 (3.1) ▲
England	518 (4.2) ▲
Slovenia	516 (2.1) ▲
Hungary	514 (3.8) ▲
Norway (9)	512 (2.3) ▲
2 Lithuania	511 (2.8) ▲
3 Israel	511 (4.1) ▲
Australia	505 (3.1)
Sweden	501 (2.8)
TIMSS Scale Centerpoint	**500**
2 Italy	494 (2.5) ▼
Malta	494 (1.0) ▼
† New Zealand	493 (3.4) ▼
Malaysia	465 (3.6) ▼
United Arab Emirates	465 (2.0) ▼
Turkey	458 (4.7) ▼
Bahrain	454 (1.4) ▼
1 2 Georgia	453 (3.4) ▼
Lebanon	442 (3.6) ▼
ψ Qatar	437 (3.0) ▼
ψ Iran, Islamic Rep. of	436 (4.6) ▼
Thailand	431 (4.8) ▼
ψ Chile	427 (3.2) ▼
ψ Oman	403 (2.4) ▼
ψ Kuwait	392 (4.6) ▼
ψ Egypt	392 (4.1) ▼
ψ Botswana (9)	391 (2.0) ▼
⁎ Jordan	386 (3.2) ▼
⁎ Morocco	384 (2.3) ▼
⁎ South Africa (9)	372 (4.5) ▼
⁎ Saudi Arabia	368 (4.6) ▼
Benchmarking Participants	
ǂ Quebec, Canada	543 (3.9) ▲
Ontario, Canada	522 (2.9) ▲
Dubai, UAE	512 (2.1) ▲
1 Florida, US	493 (6.4)
Norway (8)	487 (2.0) ▼
Abu Dhabi, UAE	442 (4.7) ▼
† ⁎ Buenos Aires, Argentina	396 (4.2) ▼

Mathematics Achievement Distribution

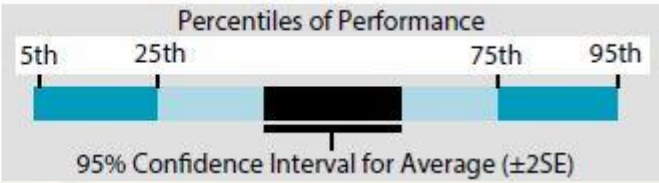
100 200 300 400 500 600 700 800

Secondary 2 Mathematics (first 15 countries)

Country	Average Scale Score	Mathematics Achievement Distribution
[2] Singapore	621 (3.2) ▲	
Korea, Rep. of	606 (2.6) ▲	
Chinese Taipei	599 (2.4) ▲	
Hong Kong SAR	594 (4.6) ▲	
Japan	586 (2.3) ▲	
Russian Federation	538 (4.7) ▲	
Kazakhstan	528 (5.3) ▲	
[1] † Canada	527 (2.2) ▲	
Ireland	523 (2.7) ▲	
† United States	518 (3.1) ▲	
England	518 (4.2) ▲	
Slovenia	516 (2.1) ▲	
Hungary	514 (3.8) ▲	
Norway (9)	512 (2.3) ▲	
[2] Lithuania	511 (2.8) ▲	

▲ Country average significantly higher than the centerpoint of the TIMSS 8th grade scale

▼ Country average significantly lower than the centerpoint of the TIMSS 8th grade scale

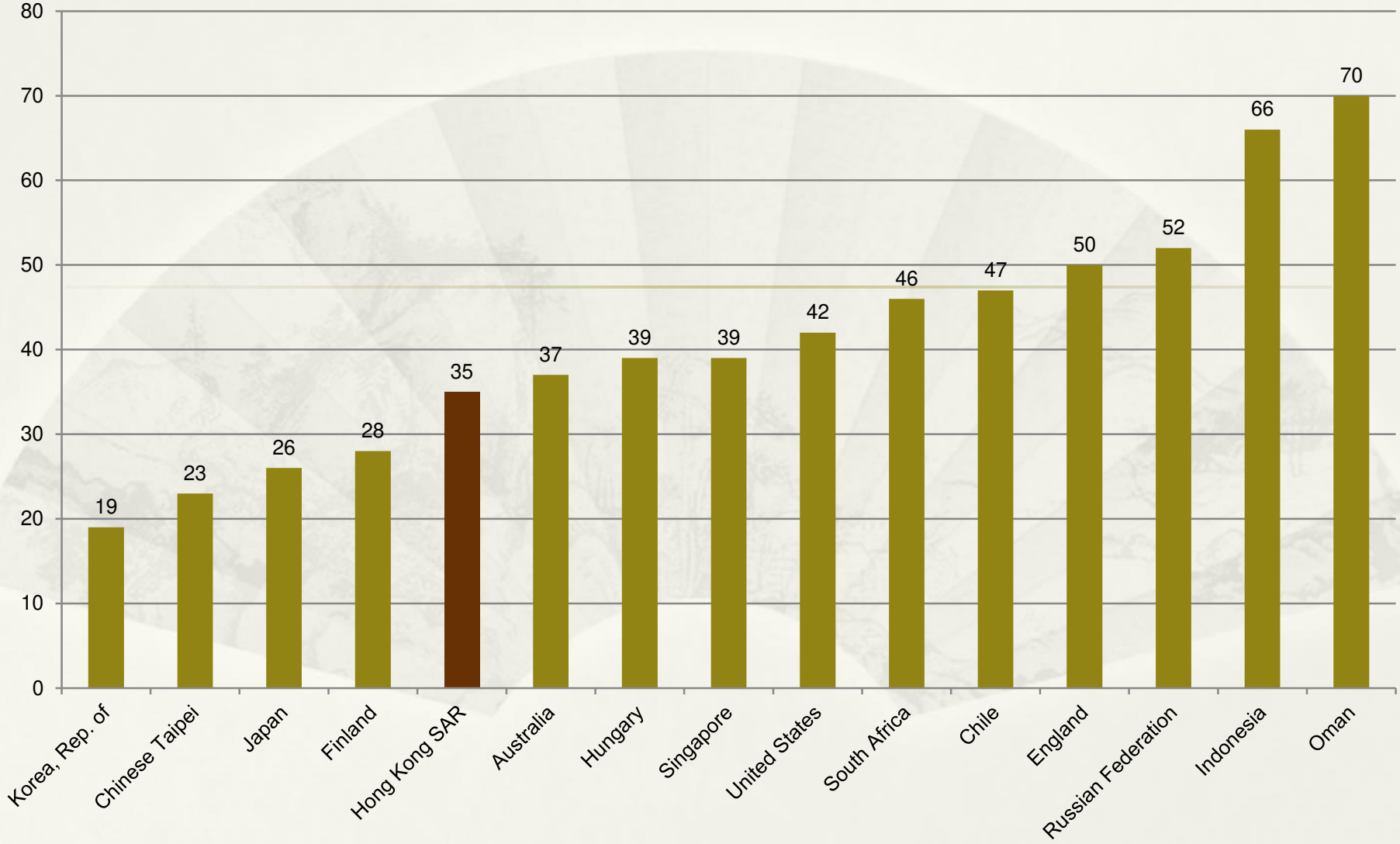


Are Hong Kong students strong or weak in mathematics?

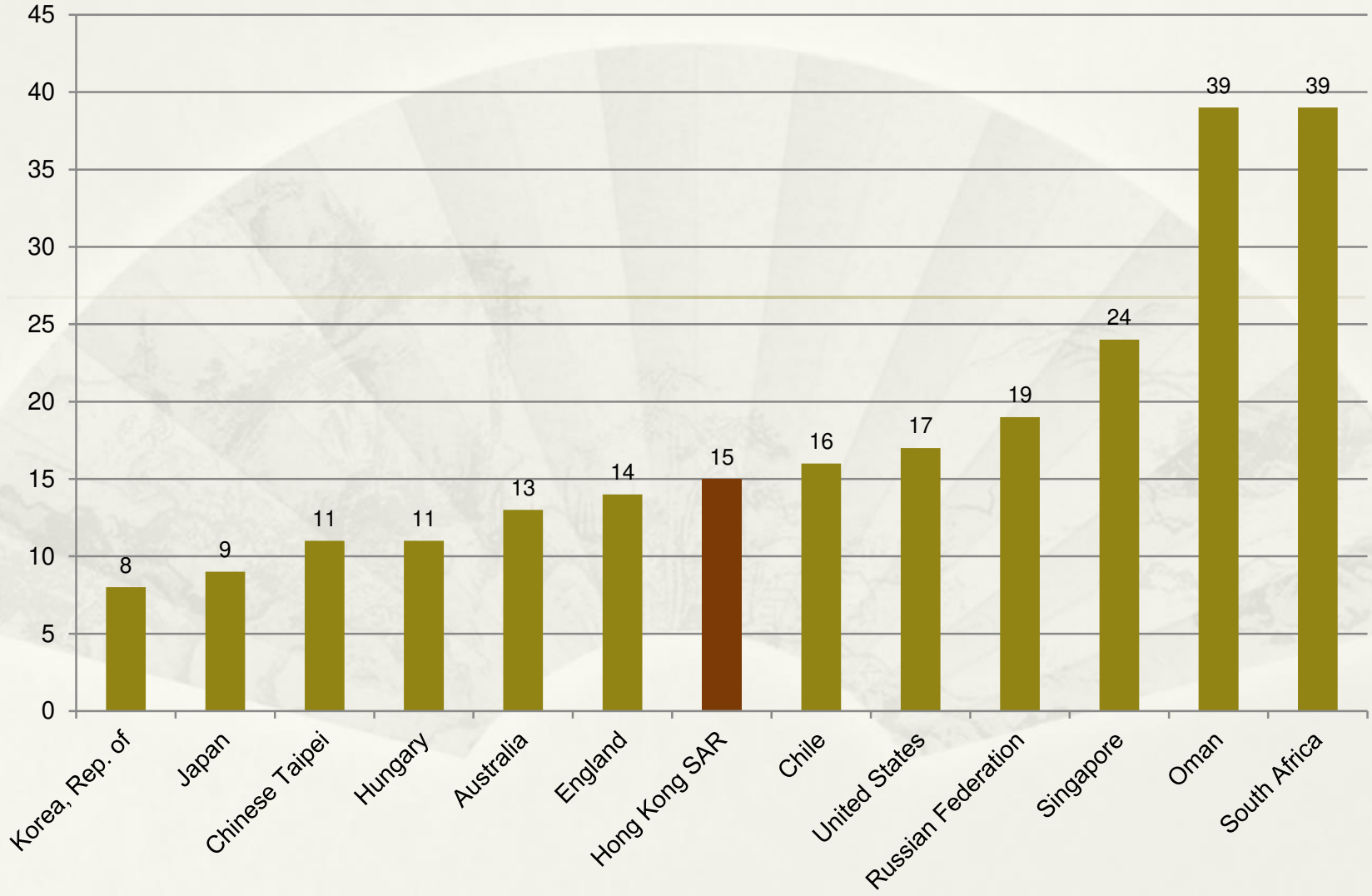
- Hong Kong students performed very well compared to students from other parts of the world
- Are the superior achievements accompanied by correspondingly positive attitudes towards mathematics and mathematics learning?

P4: Students like learning mathematics

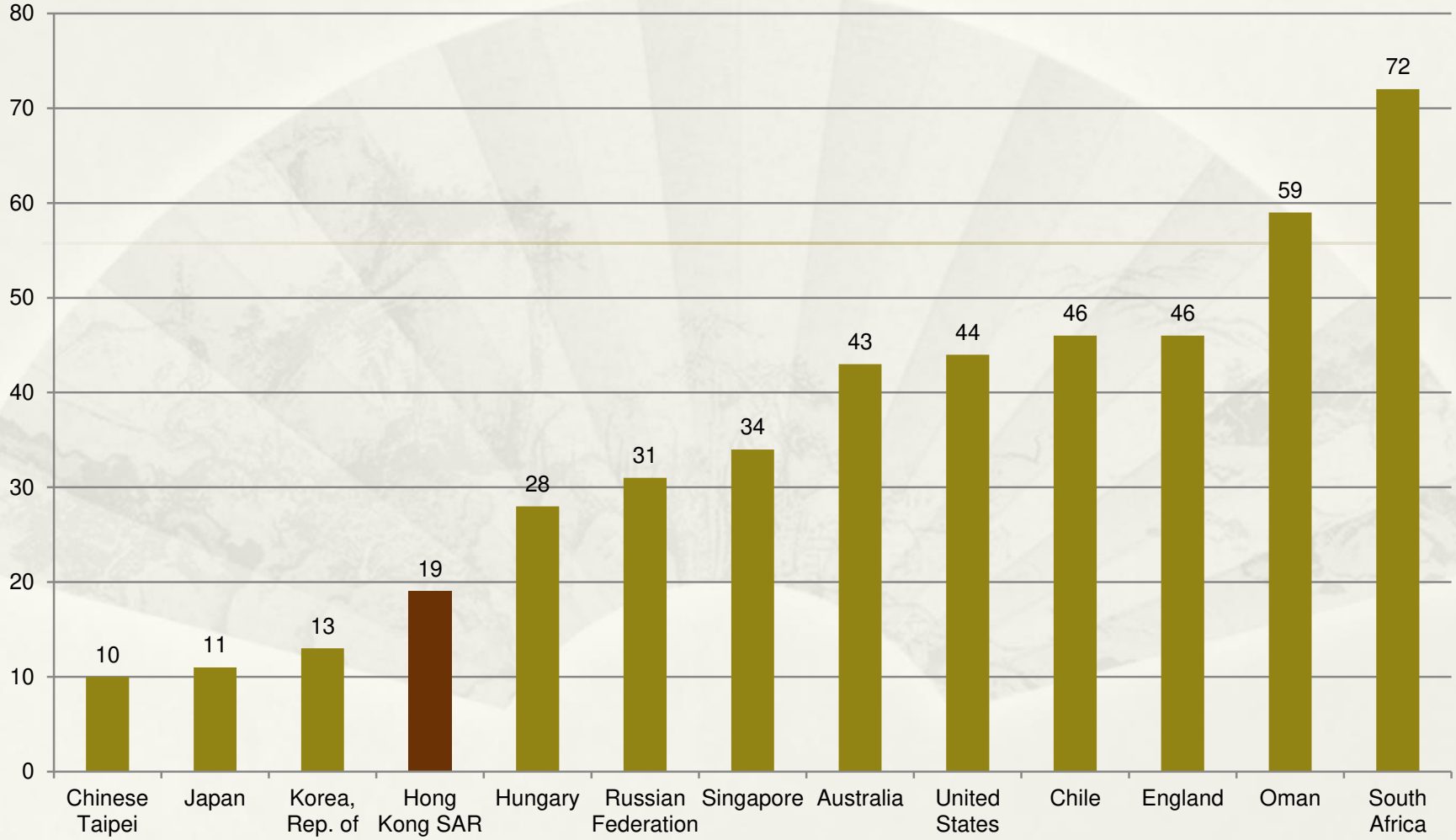
(international average = 46%)



S2: Students like learning mathematics (international average = 22%)

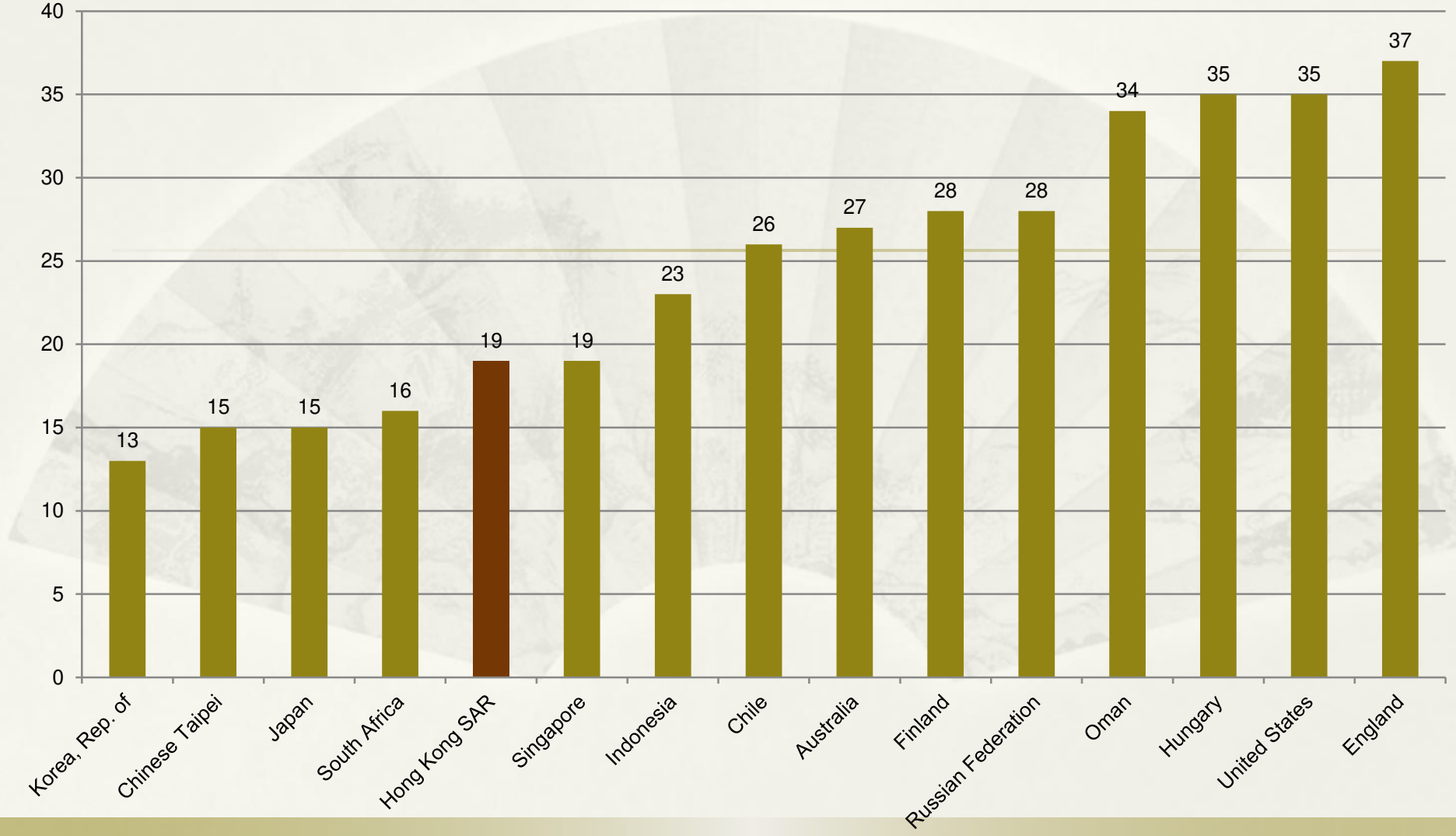


S2: Students valuing mathematics (international average = 42%)



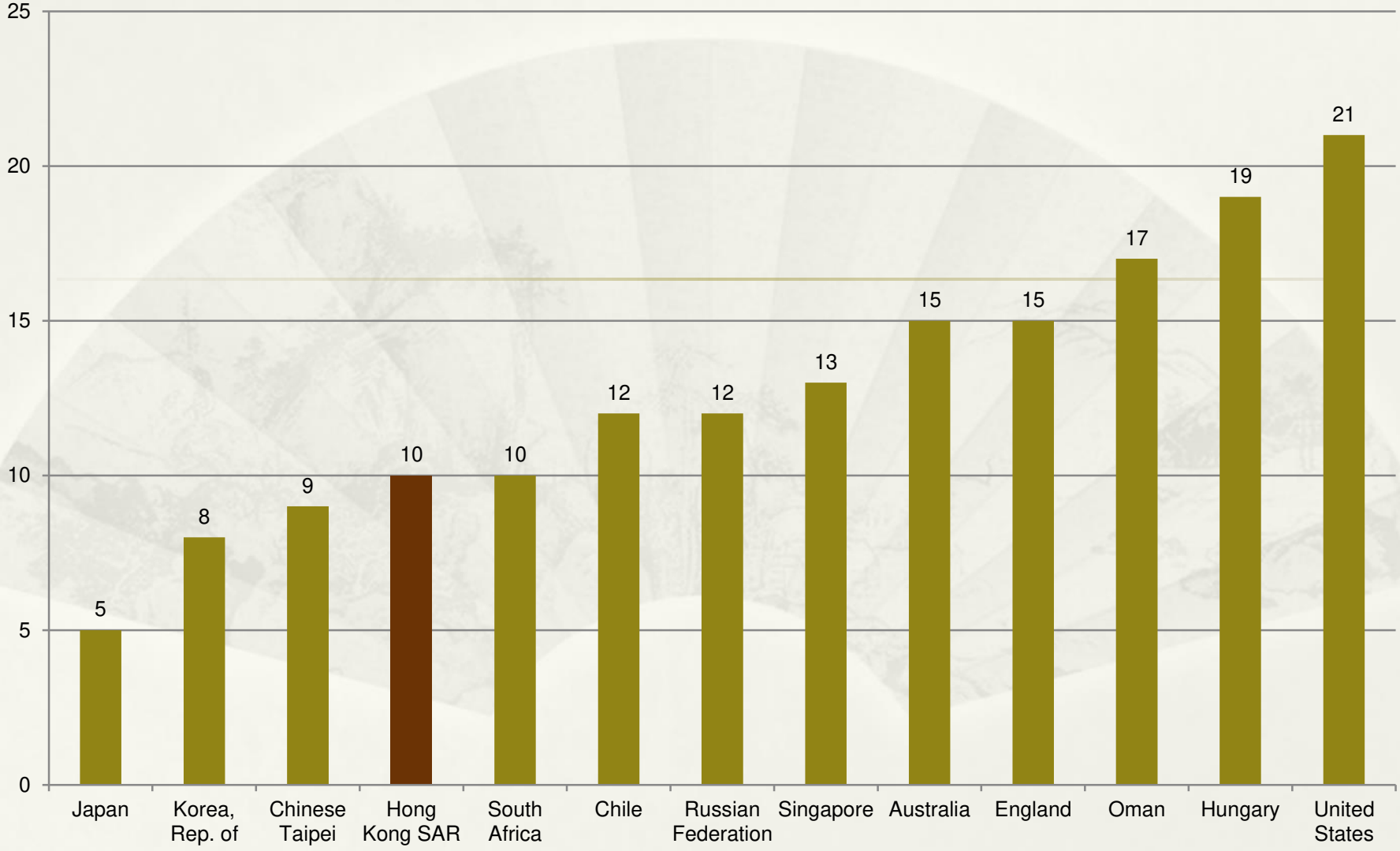
P4: Students' confidence in mathematics

(international average = 32%)



S2: Students' confidence in mathematics

(international average = 14%)



- Compared to other parts of the world, Hong Kong students perform well in mathematics, and yet have negative attitudes towards mathematics learning
- But within Hong Kong, positive attitudes are still strongly correlated with high achievement
- For life-long learning, positive attitudes may be even more important than high achievement!

Message 1

- **We should not only aim at high achievement, but should also inculcate children's positive attitudes towards mathematics**

Why do Hong Kong students have negative attitudes towards mathematics learning?

- Negative attitudes towards mathematics learning not confined to Hong Kong students
- Many students find mathematics boring
- (Did you find mathematics boring when you were a student??)
- Why do students find mathematics boring?

Research on "street mathematics"

Nunes' study (1982)

- Place: Recife, a city (population 1.5 million) on the northeastern coast of Brazil, receives large number of migrant workers from the rural areas
- 30% of the workforce is engaged in "the informal sector of the economy"
- Common in Brazil for children of street vendors to help out in their parents' business
- **Children studied**: 4 boys and 1 girls aged 9-15 with a mean age of 11.2 and ranging in level of schooling from 1st to 8th grade
- (The study was carried out in Portuguese.)

Procedure

- Children were approached by the interviewers on street corners or at markets where they worked alone or with their families
- Test items were presented in the course of a normal sales transaction in which the researcher posed as a customer
- At the end of the informal test, the children were asked to take part in a formal test, which was given on a separate occasion no more than a week later, and by the same interviewer
- Children answered a total of 63 questions on the informal test and 99 questions on the formal test, items in the formal test being based upon those in the informal test

Example 1a

R: How much is one coconut?

M: Thirty-five.

R: I'd like ten. How much is that?

M: [Pause] Three will be one hundred and five; with three more, that will be two hundred and ten. [Pause] I need four more. That is … [Pause] three hundred and fifteen … I think it is three hundred and fifty.

M can be said to have solved the following sub-items: (a) 35x10; (b) 35x3 (which may already have been known); (c) 105+105; (d) 210+105; (e) 315+35; and (f) 3+3+3+1

Example 1b

R: I'm going to take four coconuts. How much is that?

M: There will be one hundred five, plus thirty, that's one thirty-five … one coconut is thirty-five … that is … one forty!

Formal test

M solves the item 35x4, explaining out loud: "Four times five is twenty, carry the two, two plus three is five, times four is twenty." Answer written: 200

Example 2

R: What would I have to pay for six kilos [of watermelon at Cr$50.00 per kg]?

S: [Without any appreciable pause] Three hundred.

R: Let me see. How did you get that so fast?

S: Counting one by one. Two kilos, one hundred. Two hundred. Three hundred.

Formal test

Test item: A fisherman caught 50 fish. The second one caught six times the amount of fish the first fisherman had caught. How many fish did the lucky fisherman catch?

S: [Writes down 50x6 and 360 as the result, then answers] 36.

R repeats the problem and S does the computation again, writing down 860 as result. His oral response is 86.

R: How did you calculate this?

S: I did it like this. Six times six is thirty-six. Then I put it there.

R: Where did you put it? [S had not written down the number to be carried.]

S: [Points to the digit 5 in 50] That makes eighty-six [apparently adding 3 and 5 and placing this sum in the result].

R: How many did the first fisherman catch?

S: Fifty

Lessons learned

Message 2

- **To learn mathematics well, the learning has to be meaningful to the learner**
- **Otherwise, mathematics learning becomes an imitation exercise**

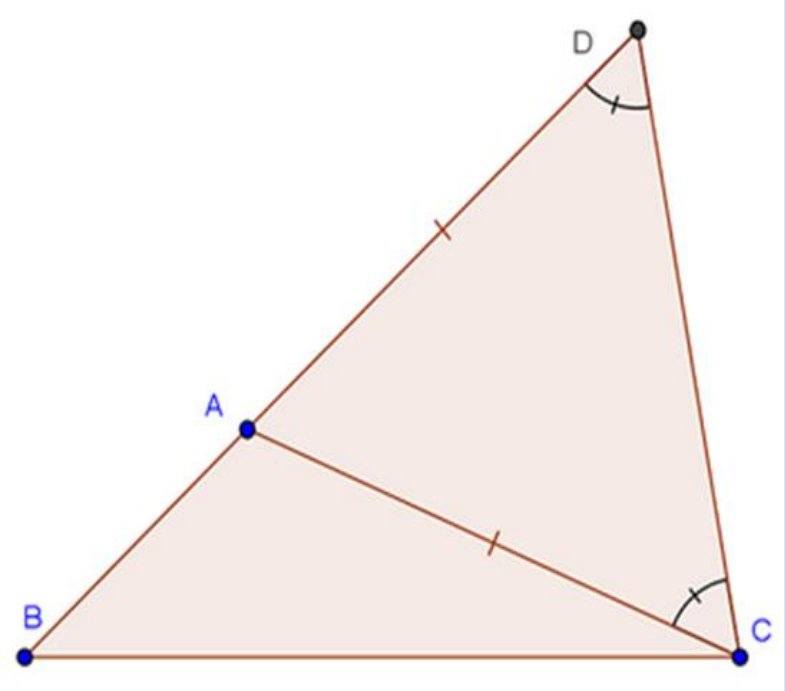
- But can all mathematics be made meaningful to children?
- If mathematics is useful, then it must be meaningful; but is mathematics useful?

Is mathematics useful?

- The example above is on arithmetic, which is very useful in our everyday life and thus relatively easy to be learned in a meaningful way
- But what about algebra, or geometry?
- For example, what's the use of geometric theorems?
- Let's look at a theorem in geometry

Theorem: The sum of any two sides of a triangle is greater than the third side.

Proof



For a ΔABC, extend BA to a point D such that DA=CA .
ΔACD is an isosceles triangle
$\therefore \angle ADC = \angle ACD$, and so
$\angle BCD > \angle BDC$

$\therefore$ BD>BC
Since BD=BA+AD and AD=AC
Thus, BA+AC>BC

A similar argument shows that AC+BC>BA and BA+BC>AC.

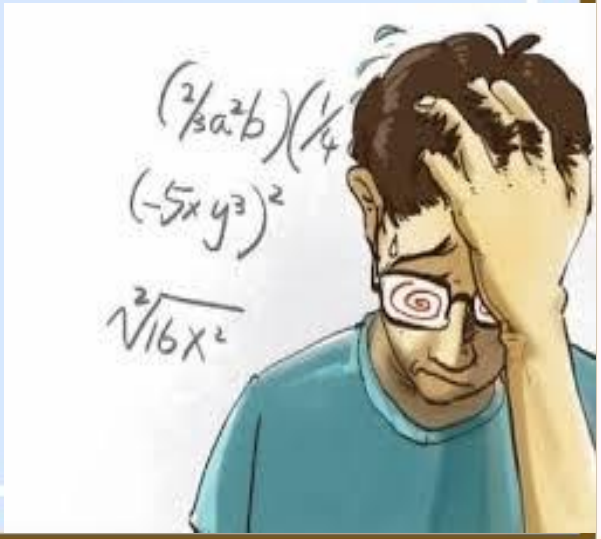
Isn't the result obvious, even without the proof?

What do we learn or gain, if any, from this theorem?

Why learn Geometry?

Why study mathematics?

Is mathematics useful?



What do we mean by "useful"?

- Mathematics (beyond the 4 rules and Statistics) is NOT useful
- Have you ever used a quadratic equation to solve your real life problem?
- Do we only learn things that are "useful"? How about music? Art?
- What do we gain through learning mathematics?
- Euclid's view

Extracts from Stobaeus

A youth who had begun to read geometry with Euclid, when he had learnt the first proposition, inquired, "What do I get by learning these things?" So Euclid called a slave and said "Give him three pence, since he must make a gain out of what he learns."

What is mathematics?

- "Mathematics is the study of quantities and shapes, and their attributes, relationships and patterns."
- Product or process?
- What is the mathematics process?
  - not just calculation
  - **abstraction** and **rigor**
- Mathematics is clear thinking and communication
- That's the precious "use" of mathematics

Mathematical thinking and everyday life

- "Silver Blaze" Sherlock Holmes (Doyle, 1892)
- A mystery about the disappearance of a famous racehorse the night before a race and the murder of the horse's trainer.

Gregory (Scotland Yard detective): "Is there any other point to which you would wish to draw my attention?"

Holmes: "To the curious incident of the dog in the night-time."

Gregory: "The dog did nothing in the night-time."

Holmes: "That was the curious incident."

No one that Sherlock Holmes spoke to in his investigation remarked that they had heard barking from the watchdog during the night when the horse was stolen.

This led Homes to conclude that the evildoer was a not a stranger, but someone the dog recognized and thus would not cause him to bark

<u>The reasoning behind</u>:

- Suppose a stranger stole the racehorse
- Then the dog should have barked
- The dog didn't bark
- Therefore the evildoer is not a stranger
- He/she must be someone the dog recognized

Deductive thinking

For example:

- A rhombus is a parallelogram
- A square is a rhombus

∴ A square is a parallelogram

Aristotle's syllogism

"All men are mortal. Socrates is a man. Therefore Socrates is mortal."

— Aristotle (?)

Are the following conclusions valid?

- All informative things are useful.
- Some websites are not useful.

∴ Some websites are not informative.

- Parent talks are boring
- Good parents attend parent talks

∴ Good parents are boring (people)

Should we occupy Central?

- Democracy is good for the livelihood of people
- Occupy Central promotes democracy
- Occupy Central affects the livelihood of people

Conclusions?

- Occupy Central does not promote democracy?
- Democracy is not good for the livelihood of people?

Should we occupy Central?

- Democracy is more than just about the livelihood of some people
- Occupy Central promotes democracy
- Occupy Central affects the livelihood of people

Conclusions?

- We need to sacrifice livelihood for the sake of democracy?
- Occupy Central is good for the livelihood of people?

Does God exist?

Proof of the existence of God

1. Only one statement in this box is true.
2. God does not exist

Proof that God does not exist

1. Only one statement in this box is true.
2. God exists

Whether God exists or not does not depend on we stupid human beings doing some paper-and-pencil "mathematical proofs"

Message 3

- Most mathematics is NOT useful in the narrow sense of usefulness
- **Mathematics is basically a way of thinking and a way of communication**
- Rigorous thinking and clear communication are extremely important in life
- In this regard, I consider mathematics (together with the Languages, perhaps) the most useful and important subject of all

How can I support my child in learning mathematics?

Message 1

- We should not only aim at high achievement, but should also inculcate children's positive attitudes towards mathematics

Message 2

- To learn mathematics well, the learning has to be meaningful to the learner

Message 3

- Mathematics is rigorous thinking and clear communication

Practice makes perfect?

- Does learning mathematics require much drill and practice?
- Practice is important, but don't over do it
- Meaningful, quality practice is more important

Speed

- Speed is good, but not very important
- Carefulness is more important than speed
- In this world of fast food culture, it is important to understand that learning well is much more important than learning fast

Important things to stress:

1. Reflection

- ask your child questions – the rationale for the work is more important than the work itself ("why?", "how did you get this answer?", "is there an another way of solving the problem?")

"How to solve it?" (Polya)

1. Understand the problem
2. Make a plan
3. Carry out the plan
4. Look back on your work. How could it be better?

2. Communication in mathematics

- Talk about mathematics
- "Tell me", "explain", "ask me questions"

3. Good learning attitude and study habit

- Process more important than the answer
- Being able to be focussed on a task and meticulous; perseverance

4. Inculcate interest and curiosity

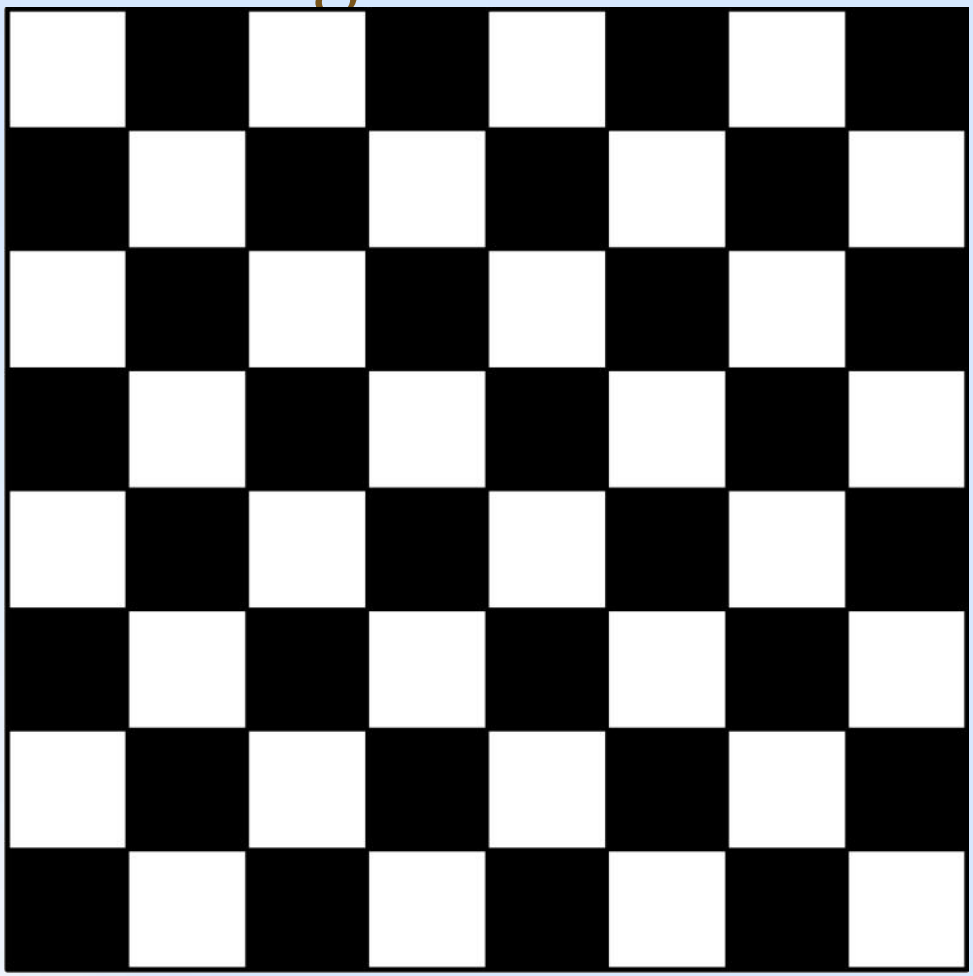
- Play mathematical games with your child, point to mathematics elements around us, ask stimulating questions to raise curiosity

5. Leave your child time and space!

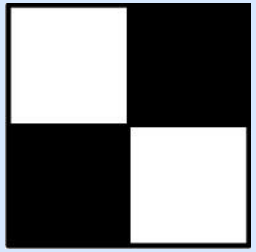
Thank you very much for your attention!

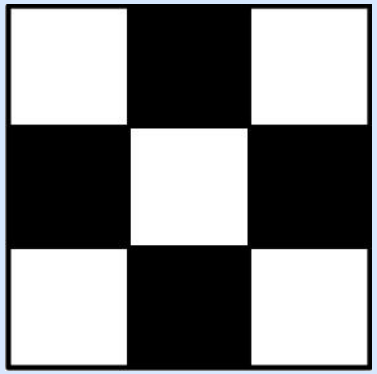
My email address:
frederickleung@hku.hk

How many squares are there in the figure below?

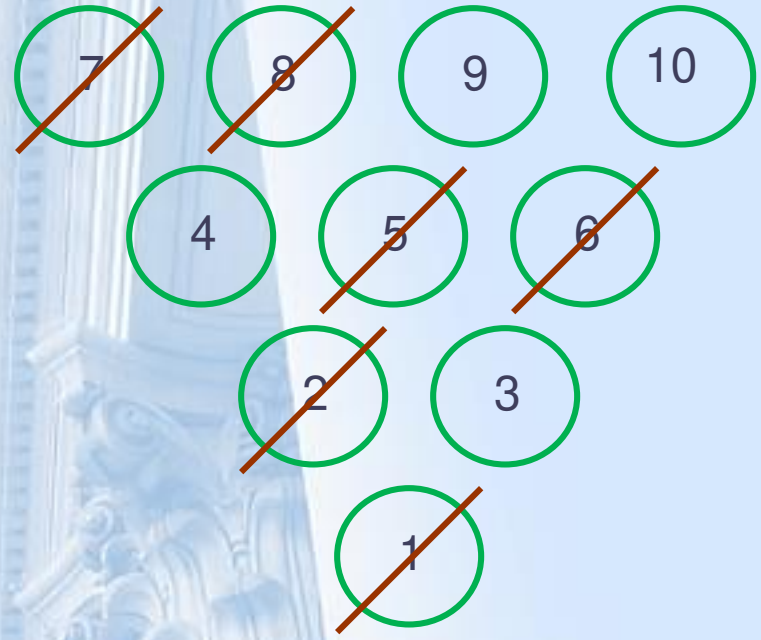


How many squares are there in the figure below





Numbers bowling



1 = 3+4-6

2 = (3×4)÷6

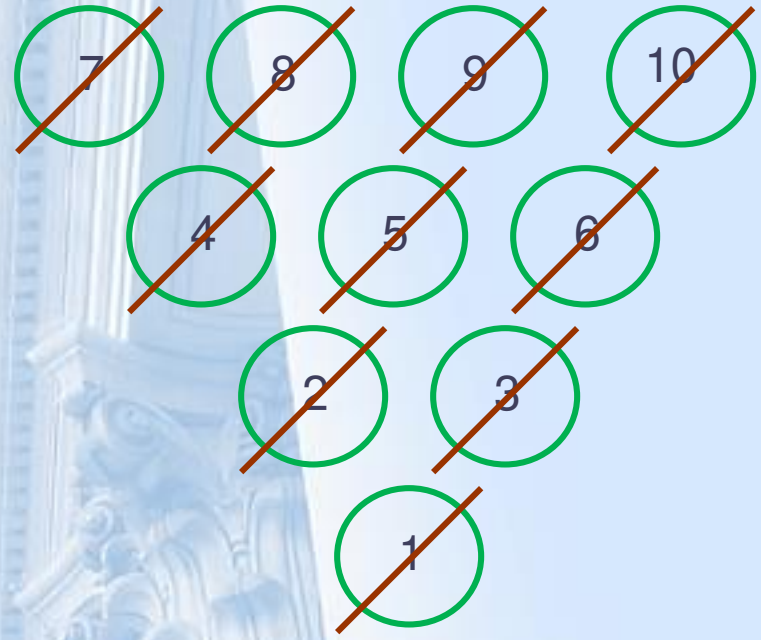
5 = 6-(4-3)

6 = 6÷(4-3)

7 = 4+6-3

8 = 4×(6÷3)

Throw three dice
e.g., the numbers are 3, 4, 6



$3 = 4-(3\div3)$

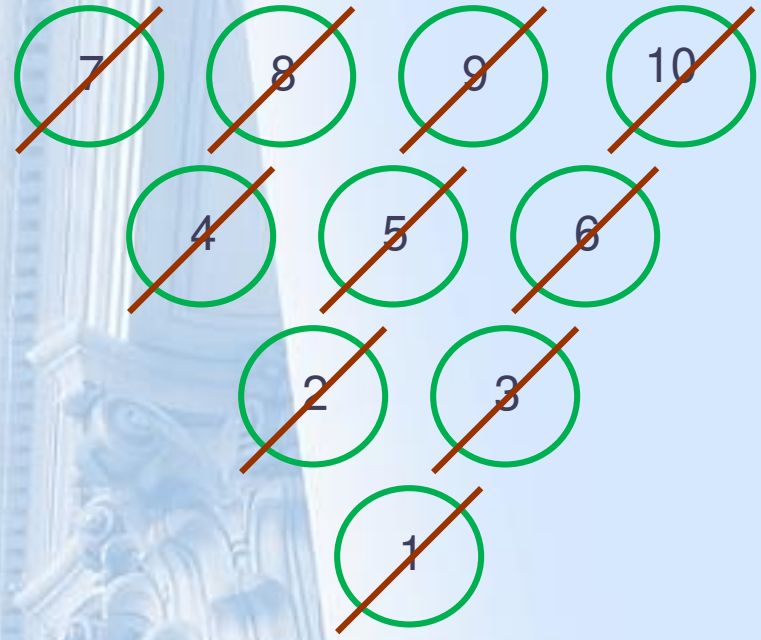
$4 = 4\times(3\div3)$

$9 = 3\times4-3$

$10 = 3+3+4$

Great!

In the second throw, the numbers are 3, 3, 4



$4 = (6/3) + \sqrt{4}$

$3 = 6 - [\sqrt{3} + \sqrt{2}]$

$9 = 3 + 4 + [\sqrt{6}]$

$10 = 3 \times 4 - [\sqrt{6}]$

Fabulous!

e.g., the three numbers are: 3, 4, 6